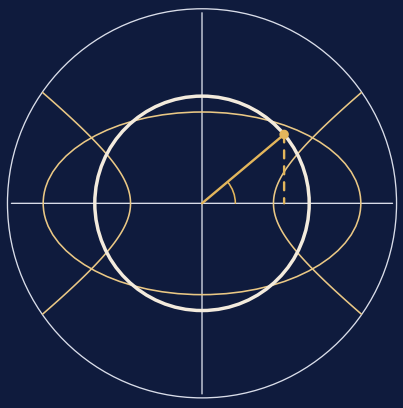
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XI

Mathematics for Class XI · CBSE

Learn · Explore · Test

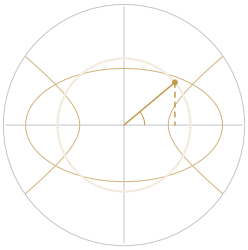
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XI

Mathematics for Class XI · CBSE

Theory · Quick Revision · Concept Exploration Exercises · Learning Progress Assignments

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Preface

Mathactive XI is part of the **Mathemission Book Series**, alongside *Mathactive XII*. It grew out of the notes, exercises, assignments and tests I have written and used with my Class XI students over several years. It follows the current CBSE syllabus and the NCERT order of chapters, and, being an e-book, it is fully linked: contents, cross-references and question–solution pairs are all one click apart. It is written to be *read*, not only consulted: every chapter builds the ideas first, then lets you test them in steps of increasing difficulty.

How each chapter is organised

Chapter opener. A short introduction and a *Key Terms* checklist. Tick the boxes as the terms become familiar.

Theory. Definitions, theorems (with proofs where they help understanding) and solved examples, each set in its own box and numbered by chapter, for example *Definition 10.2*, *Theorem 10.4*, *Example 10.4*.

Quick Revision. One table of the key results of the chapter. The "See" column gives the section where each result is explained; it is a link, so one click takes you there. A *Common Mistakes* box closes the revision.

Concept Exploration Exercises (CEE). Practice sets numbered by chapter, for example *CEE 10.1*, *CEE 10.2*, each on one part of the chapter. Answers follow each exercise, and *Brief Solutions* follow the answers, so you can check your method as well as your result.

Learning Progress Assignment (LPA). A full-length test on the chapter. See below.

The Learning Progress Assignment as a test

Each LPA follows the pattern of the CBSE board paper, so that every chapter ends with a realistic rehearsal:

Section	Questions	Type (marks each)
A	1–20	Multiple choice and Assertion–Reason (1 mark)
B	21–30	Very short answer (2 marks)
C	31–40	Short answer (3 marks)
D	41–50	Long answer (5 marks)
E	3 case studies	Case-based / source-based / passage-based (4 marks)

Treat it as a test: work section by section within the suggested time, without looking at the answers. Then mark your work against the answer key at the end of each section, and study the brief solutions, which give the key steps of each answer.

Moving between questions and solutions

Brief solutions (key steps) to every LPA are collected at the end of the e-book, chapter by chapter, so that they do not tempt you while you are attempting the test. The two are linked:

- under each LPA section heading, the link Solutions in back matter → takes you to its solutions;
- beside each solution heading, the link ← Back to questions brings you back to the questions.

The table of contents, the section references in the Quick Revision and the chapter headings are links too.

Topics outside the current syllabus

CBSE has removed some topics from the syllabus in recent years. Where a removed topic is still useful for understanding, or is needed by a few exercises, it is kept and marked with the tag Not in current CBSE syllabus next to its heading. You may skip these sections for the board examination; students who want a deeper understanding will find them worth reading. Topics that no exercise needs have been left out.

A word on the answers

Every answer in this e-book has been checked, most of them by independent computation. If you still find a mistake, or a solution that could be clearer, please write to me at `mathactiveonly@gmail.com`. Corrections will go into the next update of the e-book.

How to use this e-book

Read the theory with a pencil in hand and work each example yourself before reading its solution. Use the Quick Revision the evening before a test. Do the CEEs as you finish each part of the chapter, and the LPA only when the whole chapter is done. When you get a question wrong, go back to the section it came from, not just to the solution.

Stay Mathactive!

Ashutosh Kumar (AK)
Mathemission | www.mathemission.in
November 2026

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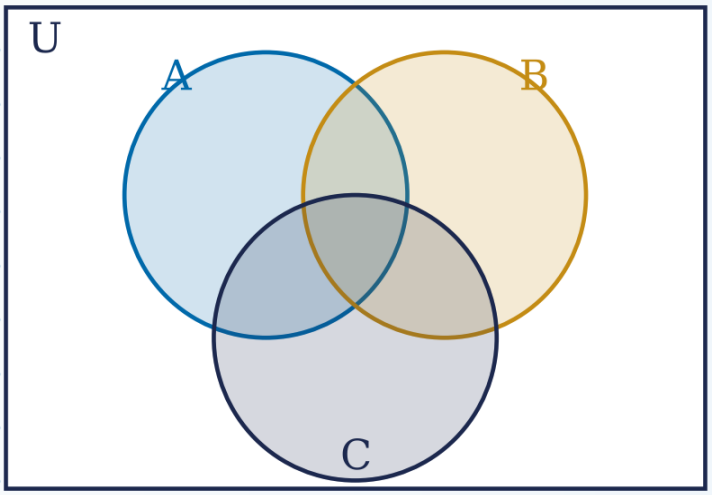
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1 Chapter Sets

A set is a well-defined collection of objects, and the language of sets runs through the whole of mathematics: domains and ranges of functions, solution sets of equations and inequalities, sample spaces in probability. This chapter fixes that language: how to describe a set, its subsets and intervals, and the operations of union, intersection, difference and complement, pictured with Venn diagrams.

Key Terms

- ☐ Roster and set-builder form
- ☐ Empty, finite and infinite sets
- ☐ Subsets and intervals
- ☐ Universal set and Venn diagrams
- ☐ Union, intersection, difference
- ☐ Complement and De Morgan's laws

1.1 Sets and their Representations

Definition 1.1 Set

A **set** is a *well-defined* collection of distinct objects. The objects are its **elements** (or members); we write $a \in A$ when a belongs to A and $b \notin A$ when it does not.

"Well defined" means two things: no object is listed twice, and for any object we can decide without doubt whether it belongs to the collection. So the collection of beautiful places in India is *not* a set, but the collection of Indian cities with a population above three million is.

Definition 1.2 Roster and Set-builder Forms

- **Roster form:** the elements are listed, separated by commas, inside braces. $A = \{2, 4, 6, 8\}$.
- **Set-builder form:** a variable, a colon (read "such that"), and the property every element has. $A = \{x : x = 2n,\ n \in \mathbb{N},\ n \leq 4\}$.

The roster form of a set is unique up to the order of its elements; a set can have many set-builder forms.

The set of	Description	Symbol
Natural numbers	$\{1, 2, 3, \ldots\}$	$\mathbb{N}$
Integers	$\{0, \pm1, \pm2, \ldots\}$	$\mathbb{Z}$
Rational numbers	$\left\{\frac{p}{q} : p, q \in \mathbb{Z},\ q \neq 0\right\}$	$\mathbb{Q}$
Real numbers	all points of the number line	$\mathbb{R}$
Positive reals	$\{x : x \in \mathbb{R},\ x > 0\}$	$\mathbb{R}^+$
Irrational numbers	$\{x : x \in \mathbb{R},\ x \notin \mathbb{Q}\}$	$\mathbb{T}$

1.1: Standard sets of numbers

Example 1.1

Write $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{3, 8, 15, 24, 35\}$ in the other form.

Solution: $x^2 - 5x + 6 = (x - 2)(x - 3)$, so $A = \{2, 3\}$. The elements of B are $2^2 - 1, 3^2 - 1, \ldots, 6^2 - 1$, so $B = \{x : x = n^2 - 1,\ n \in \mathbb{N},\ 2 \leq n \leq 6\}$.

1.2 Types of Sets

Definition 1.3 Empty, Singleton, Finite and Infinite Sets

- The **empty set** (null set) ϕ has no elements, e.g. $\{y : y^2 + 1 = 0,\ y \in \mathbb{R}\}$. There is only one empty set.
- A **singleton** set has exactly one element, e.g. $\{x : x \text{ is an even prime}\} = \{2\}$.
- A set is **finite** if its elements can be counted to an end; otherwise it is **infinite**. The number of elements of a finite set A is its **cardinality** $n(A)$.

Definition 1.4 Equal and Equivalent Sets

A and B are **equal**, $A = B$, if they have exactly the same elements. They are **equivalent** if $n(A) = n(B)$. Equal sets are equivalent; equivalent sets need not be equal.

For example, the letters of LIVES and of EVILS form the same set $\{L, I, V, E, S\}$, while $\{1, 2, 3\}$ and $\{a, b, c\}$ are equivalent but not equal.

1.3 Subsets and Intervals

Definition 1.5 Subset, Proper Subset, Universal Set

- $B \subseteq A$ (B is a **subset** of A) if every element of B is in A: $x \in B \Rightarrow x \in A$.
- $B \subset A$ is a **proper** subset if, in addition, A has an element not in B.
- A **universal set** U is a set containing all the sets under discussion.

ϕ and A itself are the *trivial* subsets of A. Two sets are equal exactly when each is a subset of the other: $A = B \iff A \subseteq B$ and $B \subseteq A$.

Theorem 1.1 Number of Subsets

A set with n elements has 2^n subsets, $2^n - 1$ proper subsets, and $2^n - 2$ non-empty proper subsets.

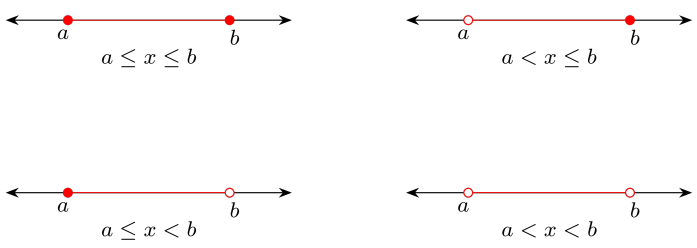
Proof: To form a subset, decide for each of the n elements whether to include it: $2 \times 2 \times \cdots \times 2 = 2^n$ choices. Leaving out A itself gives $2^n - 1$; leaving out ϕ as well gives $2^n - 2$. ■

Definition 1.6 Intervals

For real numbers $a < b$:

- closed $[a, b] = \{x : a \le x \le b\}$; open $(a, b) = \{x : a < x < b\}$;
- half-open $[a, b) = \{x : a \le x < b\}$ and $(a, b] = \{x : a < x \le b\}$;
- unbounded, e.g. $[a, \infty) = \{x : x \ge a\}$, $(-\infty, b) = \{x : x < b\}$.

The **length** of each bounded interval is $b - a$.



Intervals between two points

Example 1.2

Write $A = \{x \in \mathbb{R} : 2x - 1 \geq 0\}$ and $B = \{x \in \mathbb{R} : x^2 \leq 9\}$ as intervals. Is $B \subseteq A$?

Solution: $A = \left[\frac{1}{2}, \infty\right)$ and $B = [-3, 3]$. Since $-3 \in B$ but $-3 \notin A$, B is *not* a subset of A.

Power Set

Not in current CBSE syllabus

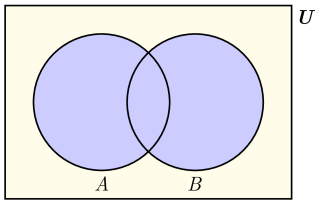
The set of all subsets of A is its **power set** $P(A)$. If $n(A) = m$ then $n(P(A)) = 2^m$. For $A = \{1, 2\}$, $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$.

1.4 Venn Diagrams and Operations on Sets

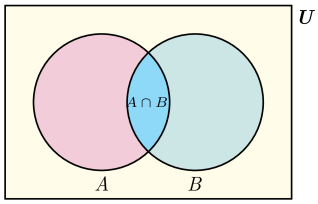
In a **Venn diagram** the universal set is a rectangle and its subsets are circles inside it.

Definition 1.7 Union, Intersection, Difference, Complement

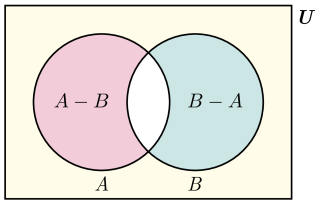
- **Union:** $A \cup B = \{x : x \in A \text{ or } x \in B\}$ (in at least one of them).
- **Intersection:** $A \cap B = \{x : x \in A \text{ and } x \in B\}$. If $A \cap B = \phi$, A and B are **disjoint**.
- **Difference:** $A - B = \{x : x \in A,\ x \notin B\} = A \cap B'$.
- **Complement:** $A' = U - A = \{x \in U : x \notin A\}$.



$A \cup B$



$A \cap B$



$A - B$

The **symmetric difference** $A \,\Delta\, B = (A - B) \cup (B - A)$ contains the elements in exactly one of the two sets.

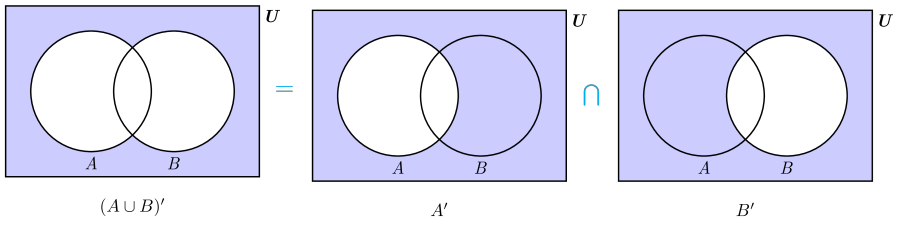
***Laws:* Laws of the Algebra of Sets**

- Idempotent: $A \cup A = A$, $A \cap A = A$. Identity: $A \cup \phi = A$, $A \cap U = A$.
- Commutative and associative laws for $\cup$ and $\cap$.
- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- Complement: $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$, $\phi' = U$, $U' = \phi$.
- **De Morgan:** $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$.
- If $B \subseteq A$: $A \cup B = A$, $A \cap B = B$, $B - A = \phi$.

Theorem 1.2 De Morgan's Law

For subsets A, B of U, $(A \cup B)' = A' \cap B'$.

Proof: $x \in (A \cup B)' \iff x \notin A \cup B \iff x \notin A$ and $x \notin B \iff x \in A'$ and $x \in B' \iff x \in A' \cap B'$. Each set is contained in the other, so they are equal. ■



$(A \cup B)'$ A' B'

$(A \cup B)' = A' \cap B'$

Example 1.3

$U = \{1, 2, \ldots, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$. Verify $(A \cap B)' = A' \cup B'$ and find $A \Delta B$.

Solution: $A \cap B = \{3, 4\}$, so $(A \cap B)' = \{1, 2, 5, 6, 7, 8, 9\}$. $A' = \{5, 6, 7, 8, 9\}$ and $B' = \{1, 2, 7, 8, 9\}$, whose union is the same set. $A \Delta B = \{1, 2\} \cup \{5, 6\} = \{1, 2, 5, 6\}$. ⛶

Example 1.4

Prove that $A - (B \cup C) = (A - B) \cap (A - C)$.

Solution: $A - (B \cup C) = A \cap (B \cup C)' = A \cap (B' \cap C') = (A \cap B') \cap (A \cap C') = (A - B) \cap (A - C)$, using De Morgan's law and $A = A \cap A$. ⛶

1.5 Counting Elements: Union and Intersection

Not in current CBSE syllabus

Theorem 1.3 Cardinality Formulas

For finite sets,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B),$$

$$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C).$$

Also $n(A - B) = n(A) - n(A \cap B)$ and $n(A') = n(U) - n(A)$. For disjoint A, B: $n(A \cup B) = n(A) + n(B)$.

In a two-set Venn diagram the four regions are *only A*, *both*, *only B* and *neither*; most survey problems are solved by filling these regions from the inside out.

Example 1.5

In a class of 40 students, 25 play cricket, 18 play football and 7 play neither. How many play both, and how many play exactly one game?

Solution: $n(C \cup F) = 40 - 7 = 33$, so $n(C \cap F) = 25 + 18 - 33 = 10$. Exactly one game: $33 - 10 = 23$ (that is, 15 only cricket and 8 only football).

Quick Revision

Key Results

Result	See
$n(A) = m \Rightarrow 2^m$ subsets, $2^m - 1$ proper, $2^m - 2$ non-empty proper	Thm 1.1
$A = B \iff A \subseteq B$ and $B \subseteq A$; $\phi \subseteq A$ for every A	§1.3
$A - B = A \cap B'$; $A \cup A' = U$, $A \cap A' = \phi$	§1.4
$(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$	Thm 1.2
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	§1.4
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$; $n(A - B) = n(A) - n(A \cap B)$	Thm 1.3
$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$	Thm 1.3

Which Tool When?

The question says...	Use
"Write in roster form"	Solve the condition and list each element once
"Is it a set?"	Can membership be decided without opinion?
Prove two sets equal	Show each is a subset of the other, or use the laws
Survey with "both", "neither", "only"	Venn diagram, fill regions from the centre outwards

***Watch out:* Common Mistakes**

- $\{\phi\}$ is not empty: it has one element.
- $\in$ relates an element to a set; $\subseteq$ relates two sets.
- $n(A-B) = n(A) - n(A \cap B)$, not $n(A) - n(B)$.
- Equivalent sets (same size) need not be equal.
- Repeated letters or numbers are listed only once in roster form.

Concept Exploration Exercise 1.1

1. Which of the following would be a set? And why?

(i) The collection of all the bowlers who have taken more than 100 wickets in One-day cricket.

(ii) The collection of all the talented batsmen of Indian Cricket.

(iii) The list of prime numbers.

(iv) The collection of lucky numbers.

2. Let S be the set of the first six natural numbers that are one less than a multiple of 5.

(i) Write S in the roster form.

(ii) Write S in a set-builder form.

(iii) Which of the following is/are **True** about S.

(A) $30 \in S$ **(B)** $20 \notin S$

(C) $19 \in S$ **(D)** $9 \notin S$

3. Consider two sets

$$A = \{x : (x^2 - 6x + 8)(x - 8)(x - 16) = 0,\ x \in \mathbb{N}\}, \text{ and}$$

$$B = \{y \,|\, y = 2^n,\ n \leq 4,\ n \in \mathbb{N}\},$$

where $\mathbb{N}$ denotes the set of natural numbers. Write the sets A and B in roster form. Do the sets A and B have same elements?

4. Let $E = \{2, 4, 6, 8, \cdots\}$ be the roster form of the set of even natural numbers. Give any two possible set-builder forms for the set E.

5. Let M be a set defined as $M = \{m : (3m - 2)$ is a prime number, for $m \leq 20,\ m \in \mathbb{N}\}$.

(i) Write M in the roster form.

(ii) Write in roster form the set $S = \{m : m \in M$ and $m^2 - 2$ is also a prime number$\}$.

(iii) Construct a set P, with $p \in P$, so that $p = 3m - 2$, where $m \in M$.

6. Consider three sets,

$$A = \{a : a = (n-1),\ 2 \le n \le 11,\ n \in \mathbb{N}\}$$
$$B = \{b : b = (a+1)^2,\ a \in A\}$$
$$C = \{c : c^2 = b,\ b \in B,\ c \text{ is an integer}\}.$$

Which of the following **is/are True**?

(A) $n(A) = n(B) + 1$ **(B)** $n(A) + n(B) = n(C)$

(C) $2\,n(B) = n(C)$ **(D)** $n(B) < n(A)$

7. For the sets, $S = \{x : x = 3n + 1,\ x < 50,\ n \in \mathbb{N}\}$ and

$$P = \{p : p = (2^n - 1) \text{ is a prime number less than } 100, \text{ where } n \in \mathbb{N}\},$$

which of the following **is/are True**?

(A) $n(P) + n(S) = 19$ **(B)** $n(S) = 5\,n(P) + 1$

(C) $n(S) = (n(P) + 1)^2$ **(D)** $n(S) - n(P) = 13$

8. Let A be the set of letters in the word **LIVES**, B be the set of the letters of the word **EVILS**, and C be that of the word **VEILS**. Also, set V be the set of vowels of the English alphabets. Write each set in the roster form. Comment on the elements and cardinality of these sets.

9. Let the sets A, B and C be defined as follow

$$A = \{x : 3x^2 + 4x + 1 = 0,\ x \in \mathbb{R}^+\}$$
$$B = \{y : y^2 + 1 = 0,\ y \in \mathbb{R}\}$$
$$C = \{r : r^2 - 2 = 0,\ r \in \mathbb{Q}\}.$$

Let N be a set of 'numbers of elements' in the sets A, B, C. Then which of the following **is/are** true?

(A) $n(A) = n(B) = n(C)$

(B) $A = B = \phi$, and C is a singleton set

(C) N is a singleton set

(D) $N(\neq \phi)$ is non-singleton finite set

10. Let the sets A, and B be defined as follow

$$A = \{y : 2^y + 1 = 17,\ y \in \mathbb{N}\}$$
$$B = \left\{\frac{1}{r} : r \le 100,\ r \in \mathbb{N}\right\}$$

Let N be a set of 'numbers of elements' in the sets A, and B. Then which of the following **is/are** true?

(A) $n(A) + n(B) = 100$ **(B)** A is a singleton set

(C) $n(N) = n(A) + n(B)$ **(D)** $n(N) + 99 = n(A) + n(B)$

11. Write the following sets in a set-builder form?

(a) E, a set of all even prime numbers.

(b) $A = \{2, 5, 10, 17, 26, 37, 50\}$

(c) S, a set of odd numbers less than 20.

(d) $B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

(e) $C = \{15, 22, 29, 36, 43\}$

(f) $D = \{9, 99, 999, 9999, 99999\}$

(g) $F = \{\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}\}$

12. Match the statements in **Column I** with those in **Column II**.

Column I	**Column II**
(A) The set of days of a week is	**(p)** a finite set
(B) The set of lines perpendicular to the $x-$axis is	**(q)** an infinite set
(C) Set $C = \{y : y < 3 \text{ and } y > 7,\ y \in \mathbb{Z}\}$ is	**(r)** a singleton set
(D) The set of circles centered at $(-2, 10)$, is	**(s)** the empty set
(E) The set of circles centered at the origin and have radius 6, is	**(t)** a set of cardinality 2
(F) The set of quadratic equations with solutions -1 and 5, is	
(G) The set $G = \{x : 5x - 2 = 0, x \in \mathbb{N}\}$, is	
(H) The set $H = \{x : 9x^2 - 1 = 0, x \in \mathbb{Q}\}$, is	
(I) The set of pairs of perpendicular lines through origin, is	

ANSWERS

1. **(i)** a set
(ii) not a set
(iii) a set
(iv) not a set

2. **(i)** $S = \{4, 9, 14, 19, 24, 29\}$
(ii) $S = \{x : x = 5n - 1,\ n \leq 6,\ n \in \mathbb{N}\}$
(iii) **(B), (C)**

3. $A = B = \{2, 4, 8, 16\}$; yes

4. $E = \{x : x = 2n, n \in \mathbb{N}\}$, $E = \{y : y \in \mathbb{N} \text{ and } 2 \text{ divides } y\}$

5. **(i)** $M = \{3, 5, 7, 11, 13, 15\}$
(ii) $S = \{3, 5, 7, 13, 15\}$
(iii) $P = \{7, 13, 19, 31, 37, 43\}$

6. **(B), (C)**

7. **(A), (B), (C), (D)**

8. $A = B = C = \{L, I, V, E, S\}$, $V = \{a, e, i, o, u\}$; A, B, C are equal, each with 5 elements
9. **(A), (C)**
10. **(B), (D)**
11. **(a)** $E = \{x : x \text{ is an even prime number.}\}$
    (b) $A = \{x : x = n^2 + 1,\ n \leq 7,\ n \in \mathbb{N}\}$
    (c) $S = \{s : s = 2n - 1,\ n \leq 8,\ n \in \mathbb{N}\}$
    (d) $B = \{p : p \text{ a prime number} < 30\}$
    (e) $C = \{c : c = 7n + 8,\ n \leq 5,\ n \in \mathbb{N}\}$
    (f) $D = \{d : d = 10^n - 1,\ n \leq 5, n \in \mathbb{N}\}$
    (g) $F = \{f : f = \frac{n}{2n-1},\ n \leq 6,\ n \in \mathbb{N}\}$
12. **(A)** $\rightarrow$ (p)
    (B) $\rightarrow$ (q)
    (C) $\rightarrow$ (s), (p)
    (D) $\rightarrow$ (q)
    (E) $\rightarrow$ (r), (p)
    (F) $\rightarrow$ (q)
    (G) $\rightarrow$ (s), (p)
    (H) $\rightarrow$ (t), (p)
    (I) $\rightarrow$ (q)

BRIEF SOLUTIONS

1. (i) and (iii) are well defined. (ii) and (iv) depend on opinion.
2. $5n - 1$ for $n = 1, \ldots, 6$.
3. $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$; with $8, 16$. 2^n for $n = 1, \ldots, 4$.
4. Any rule that produces exactly the even natural numbers.
5. $3m - 2$ is prime for $m = 3, 5, 7, 11, 13, 15$ (giving $7, 13, 19, 31, 37, 43$). $m^2 - 2 = 7, 23, 47, 119, 167, 223$; only $119 = 7 \times 17$ fails.
6. $A = \{1, \ldots, 10\}$, $B = \{4, 9, \ldots, 121\}$, $C = \{\pm 2, \ldots, \pm 11\}$: $10, 10, 20$.
7. $S = \{4, 7, \ldots, 49\}$: 16 elements. $2^n - 1 < 100$ is prime for $n = 2, 3, 5$: $P = \{3, 7, 31\}$.
8. The three words use the same five letters.
9. No positive root, no real root, no rational root: $A = B = C = \phi$, so $N = \{0\}$.
10. $2^y = 16 \Rightarrow y = 4$; $n(B) = 100$; $N = \{1, 100\}$.
11. Find the rule that generates each list: $n^2 + 1$, $2n - 1$, $7n + 8$, $10^n - 1$, $\frac{n}{2n-1}$.
12. A set with no elements is empty and finite; "all circles with a given centre" has infinitely many members.

Concept Exploration Exercise 1.2

1. Let A be the set of all real solutions of the equation $x^2 - 5x + 6 = 0$ and $B = \{y : y - 3 \geq 0,\ y \in \mathbb{R}\}$ then which of the following **is/are** true?

   (A) $A \subset B$ **(B)** $n(A) = 2$

   (C) B is an infinite set **(D)** $A = \phi$

2. Consider a set $A = \{x : x = 2n + 3,\ n < k,\ n \in \mathbb{Z}^+,\ k \in \mathbb{Q}\}$

   (i) Let $k = 4$. Write A in the roster form.

   (ii) List all the proper subsets of the set A that you found in part **(i)**.

   (iii) Find the number of non-empty proper subsets of A if $k = \dfrac{39}{5}$.

3. Write the following intervals in set builder notation. **(i)** $x \in (3, 5)$ **(ii)** $y \in (-\infty, 6]$.
   (iii) Taking $A = (3, 5)$ and $B = (-\infty, 6]$, show that $A \subset B$.

4. For two sets A and B, given $n(A) = m$ and $n(B) = k$. Also, the number of subsets of A is 30 more than that of B.

(i) Find the value of $m + k$.

(ii) Given that the difference of number of subsets of $\mathcal{P}(A)$ and $\mathcal{P}(B)$ is $2^p(2^q - 1)$, where $p, q \in \mathbb{N}$ and $\mathcal{P}(A)$ is power set of A. Find the value of $100p + 10q + 1$.

5. Let k be the number of sets that contains at least one of the numbers $2, 3, -1, 0$. Write all these sets, and hence or otherwise find k.

(i) Write the sets $B = \{x : x^2+x-2 = 0,\ x \in \mathbb{Z}\}$, and $C = \{y : y^2-3y+2 = 0,\ y \in \mathbb{N}\}$ in the roster form.

(ii) Define $A = \{x + y : x \in B \text{ and } y \in C\}$, find $n(A)$.

(iii) Write all the subsets of A.

(iv) Is $n(\mathcal{P}(A)) = k$? Why not?

6. Consider the set $A = \{\phi, \pi, \{0, 1\}\}$. State **True/False** with proper reasoning.

(i) $\phi \in \mathcal{P}(A)$

(ii) $\phi \subset \mathcal{P}(A)$

(iii) $\pi \in A$

(iv) $\{\phi, \pi\} \subset A$

(v) $\{\{0, 1\}, \pi\} \in \mathcal{P}(A)$

(vi) $\{\{0, 1\}\} \subset \mathcal{P}(A)$

(vii) $\{\{\phi\}\} \subset \mathcal{P}(A)$

(viii) $\{\{\pi\}\} \subset \mathcal{P}(A)$

(ix) $n(\mathcal{P}(A)) = 16$

(x) $n($ non-empty proper subsets of $A) = 14$.

7. Prove the following.

(i) If $A = B$ and $B = C$, then $A = C$.

(ii) If $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

8. Let $A = \{x : 3x - 1 \geq 0,\ x \in \mathbb{R}\}$ and $B = \{x : 25x^2 - 1 \geq 0,\ x \in \mathbb{R}^+\}$.

(i) Write the sets in the form of intervals.

(ii) Show that $A \subseteq B$.

ANSWERS

1. **(B), (C)**

2. **(i)** $\{5, 7, 9\}$

(ii) $\phi, \{5\}, \{7\}, \{9\}, \{5, 7\}, \{7, 9\}, \{5, 9\}$

(iii) 126

3. **(i)** $\{x : 3 < x < 5,\ x \in \mathbb{R}\}$ **(ii)** $\{y : y \leq 6,\ y \in \mathbb{R}\}$ **(iii)** Proof

4. **(i)** 6 **(ii)** 501

5. *Hint:* Except ϕ, $k = 15$

(i) $B = \{1, -2\},\ C = \{1, 2\}$

(ii) $n(A) = 4$

(iii) *Hint:* add ϕ to the same list you found before.

(iv) No.

6. **(i)** T **(ii)** T **(iii)** T **(iv)** T **(v)** T **(vi)** F **(vii)** T **(viii)** T **(ix)** F **(x)** F

8. (i) $A = [\frac{1}{3}, \infty)$, $B = [\frac{1}{5}, \infty)$ (ii) Proof