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ASSIGNMENT P4.1

Permutations, Combinations & Binomial Theorem

Focus Topics: *Distributions, inclusion-exclusion, derangements, partitions, binomial and multinomial expansions, and sums of binomial coefficients.*

Section A: Single Correct Choice

[Q1–Q4] Each question has four options (A), (B), (C) and (D). Exactly ONE option is correct.

Q 1. The coefficient of x^{12} in the algebraic expansion of

$$\left(1 - x + x^2 - x^3\right)^8$$

is equal to:

(A) 8092 **(B)** 966 **(C)** 1014 **(D)** 1120

Q 2. The total number of permutations of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ such that no two adjacent elements have a sum divisible by 3 is:

(A) 1152 **(B)** 1440 **(C)** 1728 **(D)** 2304

Q 3. The value of the finite binomial summation

$$\mathcal{S} = \sum_{r=0}^{20} \frac{(-1)^r \binom{20}{r}}{(r+1)(r+2)}$$

is equal to:

(A) $\frac{1}{462}$ **(B)** $\frac{1}{22}$ **(C)** $\frac{2}{231}$ **(D)** $\frac{1}{42}$

Q 4. The total number of positive integral solutions (x_1, x_2, x_3, x_4) satisfying the constrained linear equation

$$x_1 + x_2 + x_3 + x_4 = 24$$

such that $x_1 \leq 5$, $x_2 \leq 7$, $x_3 \leq 8$, and $x_4 \geq 2$ is:

(A) 280 **(B)** 96 **(C)** 104 **(D)** 112

Section B: One or More than One Correct

[Q5–Q9] Each question has four options (A), (B), (C) and (D). ONE OR MORE options may be correct.

Q 5. Let $S_n = \sum_{k=0}^{n} \binom{n}{k}^2$. For any integer $n \geq 1$, which of the following statements is/are correct?

(A) $S_n = \binom{2n}{n}$

(B) $\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = 0$ for all odd positive integers n.

(C) $\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = (-1)^{n/2} \binom{n}{n/2}$ for all even positive integers n.

(D) $\sum_{k=0}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}$

Q 6. A six-digit positive integer $N = \overline{d_1 d_2 d_3 d_4 d_5 d_6}$ is formed using digits from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ without repetition. Which of the following statements is/are correct?

(A) The number of such integers satisfying $d_1 < d_2 < d_3 > d_4 < d_5 < d_6$ is 1890.

(B) The number of such integers where $d_1 < d_2 < d_3$ and $d_4 > d_5 > d_6$ is 17640.

(C) The number of such integers where each digit is strictly greater than the preceding digit is 84.

(D) The number of such integers which are divisible by 4 is 15120.

Q 7. Consider the multinomial expansion of $(1 + 2x + 3x^2 + 4x^3)^6 = \sum_{k=0}^{18} a_k x^k$. Which of the following assertions is/are correct?

(A) $\sum_{k=0}^{18} a_k = 10^6$ **(B)** $\sum_{k=0}^{18} (-1)^k a_k = 0$

(C) $a_1 = 12$ **(D)** $a_2 = 78$

Q 8. Let D_n denote the number of derangements of n distinct objects. Which of the following statements is/are correct?

(A) $D_n = (n-1)(D_{n-1} + D_{n-2})$ for all $n \geq 3$.

(B) $D_n = nD_{n-1} + (-1)^n$ for all $n \geq 2$.

(C) $D_6 = 265$

(D) $\lim_{n \to \infty} \frac{D_n}{n!} = \frac{1}{e}$

Q 9. Let $E = \sum_{r=0}^{50} r \binom{100}{2r}$. Which of the following assertions is/are correct?

(A) $E = 25 \cdot 2^{99}$ **(B)** $E = 50 \cdot 2^{98}$

(C) $\sum_{r=0}^{50} \binom{100}{2r} = 2^{99}$ **(D)** $\sum_{r=0}^{49} \binom{100}{2r+1} = 2^{99}$

Section C: Numerical Answer Type

[Q10–Q15] The answer to each question is a numerical value: an integer, or a decimal rounded off to two decimal places.

Q 10. The coefficient of x^7 in the expansion of

$$(1+x)^{10} + x\,(1+x)^9 + x^2\,(1+x)^8 + \cdots + x^7\,(1+x)^3$$

is equal to _____.

Q 11. The total number of 4-element subsets $\{a, b, c, d\}$ of the set $\{1, 2, 3, \ldots, 20\}$ such that no two elements are consecutive (i.e., $|x - y| \geq 2$ for all distinct $x, y \in \{a, b, c, d\}$) is equal to _____.

Q 12. Let N be the total number of ways to distribute 12 identical apples among 4 children such that every child receives at least one apple and no child receives more than 4 apples. The value of N is equal to _____.

Q 13. The exact value of the alternating combinatorial sum

$$\mathcal{T} = \sum_{k=0}^{15} (-1)^k \binom{30}{2k}$$

is equal to _____.

Q 14. The number of permutations of the digits $1, 2, 3, 4, 5, 6, 7$ in which neither the pattern 12 nor the pattern 34 nor the pattern 56 appears as consecutive adjacent elements is equal to _____.

Q 15. The remainder when the finite binomial sum

$$\mathcal{B} = \sum_{r=1}^{100} r^3 \binom{100}{r}$$

is divided by 1000 is equal to _____.

Section D: Linked Comprehension Type

[Q16–Q17] Each passage is followed by two linked questions.

Q 16. Passage

Consider a grid in the Cartesian plane where paths consist of single unit steps in the positive x-direction (Right: $R(1,0)$) or positive y-direction (Up: $U(0,1)$). The total number of monotonic paths from $(0,0)$ to (m,n) is given by $\binom{m+n}{m}$. If paths are restricted such that they never touch or cross the diagonal barrier line $y = x+1$, the reflection principle allows exact enumerations via combinatorial subtractions.

Answer the following:

16.1 The total number of monotonic paths from $(0,0)$ to (n,n) that stay entirely on or below the diagonal line $y = x$ is equal to:

(A) $\frac{1}{n+1}\binom{2n}{n}$ **(B)** $\frac{1}{n}\binom{2n}{n}$

(C) $\binom{2n}{n} - \binom{2n}{n-2}$ **(D)** $\frac{2}{n+1}\binom{2n}{n}$

16.2 The number of monotonic paths from $(0,0)$ to $(8,8)$ that never touch the barrier line $y = x+2$ is:

(A) 6435 **(B)** 8008 **(C)** 4862 **(D)** 11440

Q 17. Passage

For each integer $n \geq 1$, define the polynomial $P_n(x) = (1+x)^n = \sum_{r=0}^{n}\binom{n}{r}x^r$. Differentiation, integration, and roots-of-unity filter substitutions yield closed-form evaluations for multivariable binomial lattices and weighted indices.

Answer the following:

17.1 The value of the weighted binomial series $\sum_{r=1}^{n} r^2\binom{n}{r}$ is:

(A) $n(n+1)2^{n-2}$ **(B)** $n(n-1)2^{n-2}$

(C) $n^2 2^{n-1}$ **(D)** $n(n+2)2^{n-2}$

17.2 For $n = 12$, the value of the cube-root filtered sum $\binom{12}{0} + \binom{12}{3} + \binom{12}{6} + \binom{12}{9} + \binom{12}{12}$ is:

(A) 1365 **(B)** 1366 **(C)** 1364 **(D)** 1368

Section E: Matrix Match Type

[Q18–Q19] Match each item in Column-I with all the correct entries in Column-II. An item may match one or more entries.

Q 18. Match each combinatorial problem in Column-I with its corresponding evaluated numerical value in Column-II.

Column I	Column II
(A) Number of non-negative integer solutions to $x_1 + x_2 + x_3 + x_4 = 10$	(p) 286
(B) Number of strictly positive integer solutions to $y_1 + y_2 + y_3 = 12$	(q) 55
(C) Number of ways to seat 6 married couples around a circular table such that men and women alternate	(r) 86400
(D) Number of functions $f : \{1, 2, 3, 4\} \to \{1, 2, 3\}$ that are surjective	(s) 36
	(t) 120

Q 19. Match each algebraic/binomial summation in Column-I with its evaluated closed-form constant in Column-II.

Column I	Column II
(A) $\sum_{r=0}^{10} \frac{\binom{10}{r}}{r+1}$	(p) $\frac{2047}{11}$
(B) $\sum_{r=1}^{10} r\binom{10}{r}$	(q) 5120
(C) $\sum_{k=0}^{5} \binom{10}{2k}$	(r) 512
(D) $\sum_{r=0}^{10} (-1)^r \binom{10}{r} 2^r$	(s) 1
	(t) 0

■

Answer Key: Assignment P4.1

ANSWER KEY

1. A
2. A
3. B
4. A
5. ABCD
6. C
7. ACD
8. ABCD
9. ABCD
10. 330
11. 2380
12. 31
13. 0
14. 3216
15. 0
16. **16.1** A
    16.2 C
17. **17.1** A
    17.2 B
18. (A) → (p) (B) → (q)
    (C) → (r) (D) → (s)
19. (A) → (p) (B) → (q)
    (C) → (r) (D) → (s)

Hints and Solutions: Assignment P4.1

Q1. (A)

Factor the expression:

$$1 - x + x^2 - x^3 = (1 - x)(1 + x^2).$$

Hence

$$(1 - x + x^2 - x^3)^8 = (1 - x)^8(1 + x^2)^8.$$

The coefficient of x^{12} is

$$\sum_{k=2}^{6}(-1)^{12-2k}\binom{8}{12-2k}\binom{8}{k} = \binom{8}{0}\binom{8}{6}+\binom{8}{2}\binom{8}{5}+\binom{8}{4}^2+\binom{8}{6}\binom{8}{3}+\binom{8}{8}\binom{8}{2}.$$

Thus the coefficient is $28 + 1568 + 4900 + 1568 + 28 = 8092$.

Q2. (A)

Classify the numbers by their residues modulo 3: there are two residues 0, three residues 1, and three residues 2. The forbidden adjacent residue pairs are 00, 12, and 21. Let $f_s(a, b, c)$ be the number of valid residue strings with a zeros, b ones, and c twos, ending in residue s. Appending the final residue gives

$$\begin{aligned} f_0(a, b, c) &= f_1(a - 1, b, c) + f_2(a - 1, b, c), \\ f_1(a, b, c) &= f_0(a, b - 1, c) + f_1(a, b - 1, c), \\ f_2(a, b, c) &= f_0(a, b, c - 1) + f_2(a, b, c - 1), \end{aligned}$$

with one-symbol strings as the initial cases and impossible states equal to zero. This recurrence gives 16 valid residue strings for $(a, b, c) = (2, 3, 3)$. For each such string, the actual numbers can be assigned in $2!\,3!\,3! = 72$ ways. Therefore the required number is $16 \times 72 = 1152$.

Q3. (B)

Use

$$\frac{1}{(r + 1)(r + 2)} = \int_0^1 x^r(1 - x)\,dx.$$

Then

$$\begin{aligned} \mathcal{S} &= \int_0^1 (1 - x)\sum_{r=0}^{20}(-1)^r\binom{20}{r}x^r\,dx \\ &= \int_0^1 (1 - x)(1 - x)^{20}\,dx = \int_0^1 (1 - x)^{21}\,dx = \frac{1}{22}. \end{aligned}$$

Q4. (A)

Set $y_1 = x_1 - 1$, $y_2 = x_2 - 1$, $y_3 = x_3 - 1$, and $y_4 = x_4 - 2$. Then

$$y_1 + y_2 + y_3 + y_4 = 19, \quad 0 \le y_1 \le 4, \quad 0 \le y_2 \le 6, \quad 0 \le y_3 \le 7, \quad y_4 \ge 0.$$

Without upper bounds, the number of solutions is $\binom{22}{3} = 1540$. By inclusion-exclusion, the single-violation counts are

$$\binom{17}{3} = 680, \qquad \binom{15}{3} = 455, \qquad \binom{14}{3} = 364,$$

and the pairwise intersection counts are

$$\binom{10}{3} = 120, \qquad \binom{9}{3} = 84, \qquad \binom{7}{3} = 35.$$

The triple intersection is empty because $5 + 7 + 8 > 19$. Hence the number of valid solutions is

$$1540 - (680 + 455 + 364) + (120 + 84 + 35) = 280.$$

Q5. (A, B, C, D)

(A) $\sum_{k=0}^{n} \binom{n}{k}^2 = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}$ by Vandermonde's Identity (A holds).

(B) Consider $(1-x)^n(1+x)^n = (1-x^2)^n$. Comparing the coefficient of x^n: LHS gives $\sum_{k=0}^{n} \binom{n}{k}(-1)^k\binom{n}{n-k} = \sum_{k=0}^{n}(-1)^k\binom{n}{k}^2$. RHS is $(1-x^2)^n = \sum_{j=0}^{n} \binom{n}{j}(-1)^j x^{2j}$, which only contains even powers of x. If n is odd, the coefficient of x^n on the RHS is 0, so the sum is 0 (B holds).

(C) If n is even, $n = 2m$, the coefficient of x^{2m} on the RHS is $(-1)^m\binom{n}{m} = (-1)^{n/2}\binom{n}{n/2}$ (C holds).

(D) Using $k\binom{n}{k} = n\binom{n-1}{k-1}$: $\sum_{k=0}^{n} k\binom{n}{k}^2 = n\sum_{k=1}^{n} \binom{n-1}{k-1}\binom{n}{n-k} = n\binom{2n-1}{n-1}$ (D holds).

Q6. (C)

(A) The inequalities in this option impose $d_1 < d_2 < d_3$ and $d_4 < d_3$, together with $d_4 < d_5 < d_6$. For any chosen set of six digits, these conditions are interleavings of two ordered chains of length three, with the extra condition that d_4 precedes d_3. There are $\binom{6}{3} = 20$ interleavings of the two chains; exactly one has d_3 before d_4. Thus the count is $19\binom{9}{6} = 19 \cdot 84 = 1596$, not 1890.

(B) The count is $\binom{9}{6}\binom{6}{3} = 84 \cdot 20 = 1680$, not 17640.

(C) Every choice of six digits gives exactly one strictly increasing arrangement, so the count is $\binom{9}{6} = 84$. This option is correct.

(D) The last two digits must be one of 12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96, giving 16 endings. The first four places can then be filled in ${}^7P_4 = 840$ ways, so the count is $16 \cdot 840 = 13440$, not 15120.

Q7. (A, C, D)

Let $P(x) = (1 + 2x + 3x^2 + 4x^3)^6 = \sum_{k=0}^{18} a_k x^k$. (A) Setting $x = 1$ gives $\sum a_k = P(1) = 10^6$, so (A) is true.

(B) Setting $x = -1$ gives $\sum(-1)^k a_k = P(-1) = (1-2+3-4)^6 = (-2)^6 = 64 \neq 0$, so (B) is false.

(C) The coefficient of x is $a_1 = \binom{6}{1}(2) = 12$, so (C) is true.

(D) The coefficient of x^2 is $a_2 = \binom{6}{2}(2)^2 + \binom{6}{1}(3) = 60 + 18 = 78$, so (D) is true.

Q8. (A, B, C, D)

(A) Derangement recurrence relation: $D_n = (n-1)(D_{n-1} + D_{n-2})$ is standard (A holds).
(B) First-order recurrence: $D_n = nD_{n-1} + (-1)^n$ is standard (B holds).
(C) Direct computation: $D_1 = 0$, $D_2 = 1$, $D_3 = 2$, $D_4 = 9$, $D_5 = 44$, $D_6 = 5(44+9) = 5(53) = 265$ (C holds).
(D) Since $D_n = n!\sum_{k=0}^{n} \frac{(-1)^k}{k!}$, we have $\lim_{n\to\infty} \frac{D_n}{n!} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = \frac{1}{e}$ (D holds).

Q9. (A, B, C, D)

Consider $(1+x)^{100} = \sum_{k=0}^{100} \binom{100}{k} x^k$. Differentiating with respect to x: $100(1+x)^{99} = \sum_{k=1}^{100} k\binom{100}{k} x^{k-1}$. Multiply by x: $100x(1+x)^{99} = \sum_{k=1}^{100} k\binom{100}{k} x^k$. Substitute $x = 1$: $100(2^{99}) = \sum_{k=1}^{100} k\binom{100}{k}$. Substitute $x = -1$: $100(-1)(0) = 0 = \sum_{k=1}^{100} k\binom{100}{k}(-1)^k$. Adding the two results: $2\sum_{r=1}^{50}(2r)\binom{100}{2r} = 100 \cdot 2^{99} \implies 4\sum_{r=1}^{50} r\binom{100}{2r} = 100 \cdot 2^{99}$. Therefore:

$$E = \sum_{r=0}^{50} r\binom{100}{2r} = \frac{100 \cdot 2^{99}}{4} = 25 \cdot 2^{99} = 50 \cdot 2^{98}$$

(A and B hold). Also, $\sum_{r=0}^{50} \binom{100}{2r} = \frac{(1+1)^{100}+(1-1)^{100}}{2} = 2^{99}$ (C holds), and odd sum is 2^{99} (D holds).

Q10. 330

Notice that the sum is a finite geometric progression of 8 terms with first term $a = (1+x)^{10}$ and common ratio $r = \frac{x}{1+x}$:

$$S = (1+x)^{10}\frac{1-\left(\frac{x}{1+x}\right)^8}{1-\frac{x}{1+x}} = (1+x)^{10}\frac{\frac{(1+x)^8-x^8}{(1+x)^8}}{\frac{1}{1+x}}$$

$$= (1+x)^3\left[(1+x)^8 - x^8\right] = (1+x)^{11} - x^8(1+x)^3$$

We want the coefficient of x^7 in S: 1. In $(1+x)^{11}$, the coefficient of x^7 is $\binom{11}{7} = \binom{11}{4} = \frac{11\times10\times9\times8}{24} = 330$. 2. In $x^8(1+x)^3$, the lowest power of x is x^8, so the coefficient of x^7 is 0. Thus, $[x^7]S = 330 - 0 = 330$.

Q11. 2380

We want to choose 4 non-consecutive numbers from $\{1, 2, \ldots, 20\}$. Let the chosen numbers be $x_1 < x_2 < x_3 < x_4$. The non-consecutive condition means: $x_2 \geq x_1 + 2$, $x_3 \geq x_2 + 2$, $x_4 \geq x_3 + 2$. Define: $y_1 = x_1$, $y_2 = x_2 - 1$, $y_3 = x_3 - 2$, $y_4 = x_4 - 3$. Then $1 \leq y_1 < y_2 < y_3 < y_4 \leq 20 - 3 = 17$. This is a one-to-one correspondence between the required subsets and choosing 4 distinct numbers from $\{1, 2, \ldots, 17\}$. Therefore:

$$\text{Number of ways} = \binom{17}{4} = \frac{17 \times 16 \times 15 \times 14}{24} = 17 \times 2 \times 5 \times 14 = 2380.$$

Q12. 31

Let x_i be the number of apples received by child $i \in \{1, 2, 3, 4\}$. We need the

number of integer solutions to:

$$x_1 + x_2 + x_3 + x_4 = 12 \quad \text{with } 1 \le x_i \le 4$$

Let $y_i = x_i - 1$. Then $0 \le y_i \le 3$, and:

$$y_1 + y_2 + y_3 + y_4 = 12 - 4 = 8$$

The problem is equivalent to finding the coefficient of t^8 in $(1 + t + t^2 + t^3)^4$:

$$(1 + t + t^2 + t^3)^4 = \left(\frac{1 - t^4}{1 - t}\right)^4 = (1 - t^4)^4 (1 - t)^{-4}$$

Expand:

$$(1 - t^4)^4 = 1 - 4t^4 + 6t^8 - \dots$$

$$(1 - t)^{-4} = \sum_{k=0}^{\infty} \binom{k+3}{3} t^k$$

The coefficient of t^8 is:

$$1 \cdot \binom{8+3}{3} - 4 \cdot \binom{4+3}{3} + 6 \cdot \binom{0+3}{3} = \binom{11}{3} - 4\binom{7}{3} + 6\binom{3}{3}$$

$$= \frac{11 \times 10 \times 9}{6} - 4\left(\frac{7 \times 6 \times 5}{6}\right) + 6(1) = 165 - 4(35) + 6 = 165 - 140 + 6 = 31.$$

Q13. 0

The sum is the real part of $(1 + i)^{30}$:

$$\mathcal{T} = \text{Re}\left(\sum_{k=0}^{30} \binom{30}{k} i^k\right) = \text{Re}\left((1 + i)^{30}\right).$$

Since $1 + i = \sqrt{2}\, e^{i\pi/4}$,

$$(1 + i)^{30} = 2^{15} e^{i15\pi/2} = -2^{15} i.$$

Its real part is zero. Therefore $\mathcal{T} = 0$.

Q14. 3216

There are $7! = 5040$ permutations in total. Let A, B, C denote the events that the consecutive patterns 12, 34, and 56 occur, respectively. Treating each specified pair as a block gives

$$|A| = |B| = |C| = 6!, \qquad |A \cap B| = |A \cap C| = |B \cap C| = 5!, \qquad |A \cap B \cap C| = 4!.$$

By inclusion-exclusion, the number containing at least one forbidden pattern is

$$3(6!) - 3(5!) + 4! = 2160 - 360 + 24 = 1824.$$

Hence the number containing none of the patterns is $5040 - 1824 = 3216$.

Q15. 0

Use $r^3 = r(r-1)(r-2)+3r(r-1)+r$ and the standard binomial sums to obtain

$$\mathcal{B} = 100 \cdot 99 \cdot 98 \cdot 2^{97} + 3(100 \cdot 99)2^{98} + 100 \cdot 2^{99}.$$

Reducing the three terms modulo 1000 gives 400, 800, and 800, respectively. Their sum is $2000 \equiv 0 \pmod{1000}$. Thus the remainder is 0.

Passage 16

16.1 (A): The required paths are counted by the Catalan number

$$C_n = \frac{1}{n+1}\binom{2n}{n}.$$

By the reflection principle, the number of paths that cross above the diagonal is $\binom{2n}{n-1}$. Therefore

$$\binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n},$$

so option (A) is correct. **16.2 (C):** There are $\binom{16}{8} = 12870$ monotonic paths from $(0,0)$ to $(8,8)$. Reflecting a path at its first contact with $y = x+2$ maps a bad path to a path from $(-2,2)$ to $(8,8)$. Such a path has 10 right steps and 6 up steps, so the number of bad paths is $\binom{16}{6} = 8008$. Hence the number that never touch the barrier is

$$12870 - 8008 = 4862.$$

Passage 17

17.1 (A): Since $r^2 = r(r-1)+r$,

$$\sum_{r=1}^{n} r^2\binom{n}{r} = n(n-1)2^{n-2} + n2^{n-1} = n(n+1)2^{n-2}.$$

Thus option (A) is correct. **17.2 (B):** The required sum is

$$\binom{12}{0} + \binom{12}{3} + \binom{12}{6} + \binom{12}{9} + \binom{12}{12} = 1 + 220 + 924 + 220 + 1 = 1366.$$

Thus option (B) is correct.

Q18 & Q19

For Q18:

- (A) By stars and bars, $\binom{10+4-1}{4-1} = \binom{13}{3} = 286$, so (A) matches (p).
- (B) The positive solutions are counted by $\binom{12-1}{3-1} = \binom{11}{2} = 55$, so (B) matches (q).
- (C) Seat the six men around the circle in 5! ways and arrange the six women in the six gaps in 6! ways; $5!\,6! = 86400$, so (C) matches (r).

- (D) By inclusion-exclusion, the number of surjections is $3^4 - \binom{3}{1}2^4 + \binom{3}{2}1^4 = 36$, so (D) matches (s).

For Q19:

- (A) $\sum_{r=0}^{10} \frac{\binom{10}{r}}{r+1} = \frac{2^{11}-1}{11} = \frac{2047}{11}$, so (A) matches (p).
- (B) $\sum_{r=1}^{10} r\binom{10}{r} = 10 \cdot 2^9 = 5120$, so (B) matches (q).
- (C) $\sum_{k=0}^{5} \binom{10}{2k} = 2^9 = 512$, so (C) matches (r).
- (D) $\sum_{r=0}^{10}(-1)^r\binom{10}{r}2^r = (1-2)^{10} = 1$, so (D) matches (s).