MATHEMISSION BOOK SERIES

गणिते रमध्वं नित्यम्

FREE

Rankers Assignment

PLUS

PART V · FREE SAMPLE

Coordinate Geometry

Assignment P5.1 with answer key and solutions

Try it like a real test: 75–90 minutes, no peeking

Liked it? The full Part and the complete e-book are on mathemission.in

ASHUTOSH KUMAR

Stay Mathactive · Mathemission

mathemission.in

ASSIGNMENT P5.1

Straight Lines, Pair of Lines & Families of Lines

Focus Topics: *Forms of a line, distances and reflections, angle bisectors, families of lines, pair of straight lines, loci, and lines meeting circles.*

Section A: Single Correct Choice

[Q1–Q4] Each question has four options (A), (B), (C) and (D). Exactly ONE option is correct.

Q 1. The foot of the perpendicular drawn from $P(1,2)$ to a variable line in the pencil

$$L_{u,v} : u(x-2)+v(y-3)=0, \qquad (u,v)\neq(0,0),$$

traces a locus whose equation is:

(A) $(x-1)(x-2)+(y-2)(y-3)=0$

(B) $(x-1)^2+(y-2)^2=1$

(C) $(x-2)^2+(y-3)^2=1$

(D) $(x-1)(y-3)+(x-2)(y-2)=0$

Q 2. Let m_1, m_2 be the roots of $m^2-am-a-1=0$. For $a=1$, the area of the triangle formed by the lines $y=m_1x$, $y=m_2x$ and $y=1$ is:

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** 1

Q 3. The equation of the pair of angle bisectors of the lines represented by

$$2x^2-5xy+2y^2=0$$

is:

(A) $x^2-y^2=0$ **(B)** $x^2-5xy+6y^2=0$

(C) $(x+y)^2=0$ **(D)** $3x^2-10xy+3y^2=0$

Q 4. A line with positive intercepts on the coordinate axes passes through $P(2,3)$. Among all such lines, the one that forms the triangle of least area with the coordinate axes is:

(A) $3x+2y=12$ **(B)** $2x+3y=13$ **(C)** $x+y=5$ **(D)** $3x+2y=6$

Section B: One or More than One Correct

[Q5–Q9] Each question has four options (A), (B), (C) and (D). ONE OR MORE options may be correct.

Q 5. Two equal sides of an isosceles triangle lie along the lines

$$7x - y + 3 = 0 \qquad \text{and} \qquad x + y - 3 = 0.$$

The third side passes through $(1, -10)$. Which of the following can be its equation?

(A) $x + 3y + 29 = 0$ **(B)** $x - 3y = 31$

(C) $3x - y = 13$ **(D)** $3x + y + 7 = 0$

Q 6. A line passes through $P(2, 3)$ and cuts an intercept of length 2 between the parallel lines

$$2x + y = 3 \qquad \text{and} \qquad 2x + y = 5.$$

Which of the following can be the line?

(A) $x = 2$ **(B)** $y = 3$

(C) $3x + 4y - 18 = 0$ **(D)** $2x + y = 7$

Q 7. The equation

$$ax^2 - 6xy + y^2 + bx + cy + d = 0$$

represents a pair of straight lines whose slopes are m and m^2. Which values can a take?

(A) -8 **(B)** 8 **(C)** 27 **(D)** -27

Q 8. The vertex A of $\triangle ABC$ lies on $3y = 4x + 15$, while side BC, of length 2, lies on $3y = 4x$. Which statements are correct?

(A) The area of $\triangle ABC$ is 3 square units.

(B) The locus of the orthocentre is $3y = 4x + 5$.

(C) The locus of the centroid is $3Y = 4X + 5$, where (X, Y) is the centroid.

(D) The length of the median through A is constant for all such triangles.

Q 9. Consider the pair of straight lines

$$6x^2 - 7xy + y^2 = 0.$$

Which statements are correct?

(A) The slopes of the two lines are 6 and 1.

(B) The two lines are perpendicular.

(C) $\tan\theta = \dfrac{5}{7}$, where θ is the acute angle between the lines.

(D) The equation factors as $(6x - y)(x - y) = 0$.

Section C: Numerical Answer Type

[Q10–Q15] The answer to each question is a numerical value: an integer, or a decimal rounded off to two decimal places.

Q 10. A line cuts the circle $x^2 + y^2 = 100$ at two points, each at a distance 4 from $P(8, 6)$. Its equation is $ax + by - 46 = 0$, where $a, b \in \mathbb{N}$. Find $a + b$. _____

Q 11. A rigid triangle ABC has side lengths $AB = 13$, $BC = 5$ and $AC = 12$. As it moves with A on the positive x-axis and B on the positive y-axis, the locus of C is a line $12x - ky = 0$. Find k. _____

Q 12. Let $A(4, -2)$, $B(-2, 4)$ and $C(5, 5)$ be the vertices of $\triangle ABC$. The perpendicular bisector of BC meets the circumcircle again at D, chosen so that A and D lie on opposite sides of BC. Find the distance from the origin to line AD, rounded to two decimal places. _____

Q 13. The chord of the circle

$$x^2 + y^2 - 4x - 6y = 0$$

joining the origin to another point on the circle subtends an angle $\tan^{-1}(7/4)$ at $(0, 6)$. The chord is one member of the pair of lines

$$3x^2 - 5xy - 2y^2 - x + ky = 0.$$

Find k. _____

Q 14. The lines represented by $x^2 - 5xy + 6y^2 = 0$ meet at an acute angle θ. Find $7 \tan\theta$. _____

Q 15. A member of the family

$$L_t : (1 + 2t)x + (2 + t)y - 5t = 0$$

is parallel to $3x + 4y - 7 = 0$. Find $5t$. _____

Section D: Linked Comprehension Type

[Q16–Q17] Each passage is followed by two linked single-correct questions.

Q 16. Passage: A Pencil of Lines

Consider the family

$$L_{a,b} : a(2x + y + 4) + b(x - 2y - 3) = 0, \qquad (a, b) \neq (0, 0).$$

It is generated by the two fixed lines $2x + y + 4 = 0$ and $x - 2y - 3 = 0$. Let I be their point of intersection and let $M = (2, -3)$.

Answer the following:

16.1 The point I is:

(A) $(-1, -2)$ **(B)** $(1, 2)$ **(C)** $(-1, 2)$ **(D)** $(2, -1)$

16.2 For the member of the family perpendicular to IM, the ratio $a : b$ is:

(A) $1 : 2$ **(B)** $1 : 1$ **(C)** $2 : 1$ **(D)** $3 : 1$

Q 17. Passage: An Intercept Family of Lines

For $-1 < t < 2$, consider the family of lines

$$L_t : (t + 1)x + (2 - t)y - 6 = 0.$$

Every member cuts the positive coordinate axes. Its x- and y-intercepts are denoted by a_t and b_t, respectively.

Answer the following:

17.1 The value of t for which the two intercepts are equal is:

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $\frac{3}{2}$

17.2 The member of the family perpendicular to $2x - y + 1 = 0$ corresponds to:

(A) $t = 0$ **(B)** $t = \frac{1}{2}$ **(C)** $t = 1$ **(D)** $t = \frac{3}{2}$

Section E: Matrix Match Type

[Q18–Q19] Match each item in Column-I with the corresponding property in Column-II.

Q 18. Match each equation in Column-I with the corresponding property in Column-II.

Column I	Column II
(A) $x^2 - 5xy + 6y^2 = 0$	(p) Coincident lines
(B) $3x^2 - 10xy + 3y^2 = 0$	(q) Perpendicular lines
(C) $x^2 - y^2 = 0$	(r) $\tan\theta = \frac{1}{7}$ for the acute angle θ
(D) $(x + y)^2 = 0$	(s) $\tan\theta = \frac{4}{3}$ for the acute angle θ
	(t) Distinct parallel lines

Q 19. Match each family in Column-I with its common point or envelope in Column-II.

Column I	Column II
(A) $a(x + y - 2) + b(x - 2y + 1) = 0$	(p) Fixed point $(1, 1)$
(B) $y = 2mx - m^2$	(q) Envelope $y = x^2$
(C) $x\cos\theta + y\sin\theta = 2$	(r) Envelope $x^2 + y^2 = 4$
(D) $a(x - 1) + b(y + 2) = 0,\ (a, b) \neq (0, 0)$	(s) Fixed point $(1, -2)$
	(t) A family of parallel lines

■

Answer Key: Assignment P5.1

ANSWER KEY

1. A
2. C
3. A
4. A
5. BD
6. AC
7. BD
8. AC
9. ACD
10. 7
11. 5
12. 3.16
13. 2
14. 1
15. 2
16. **16.1** A
16.2 B
17. **17.1** B
17.2 A
18. (A) → (r) (B) → (s)
(C) → (q) (D) → (p)
19. (A) → (p) (B) → (q)
(C) → (r) (D) → (s)

Hints and Solutions: Assignment P5.1

Q1. (A)

All lines in the family pass through $C = (2, 3)$. If $H = (x, y)$ is the foot of the perpendicular from $P = (1, 2)$ to one of these lines, then $PH \perp CH$. Hence H lies on the circle with diameter PC:

$$\overrightarrow{PH} \cdot \overrightarrow{CH} = 0 \quad \Longrightarrow \quad (x-1)(x-2) + (y-2)(y-3) = 0.$$

Thus option **(A)** is correct.

Q2. (C)

For $a = 1$, the equation of the slopes becomes

$$m^2 - m - 2 = 0 \quad \Longrightarrow \quad m_1 = 2,\ m_2 = -1.$$

The lines meet $y = 1$ at $(\frac{1}{2}, 1)$ and $(-1, 1)$. The triangle has horizontal base length $\frac{3}{2}$ and height 1, so its area is

$$\frac{1}{2} \cdot \frac{3}{2} \cdot 1 = \frac{3}{4}.$$

The answer is **(C)**.

Q3. (A)

Factor the pair of lines:

$$2x^2 - 5xy + 2y^2 = (2x - y)(x - 2y) = 0.$$

The two line equations have equal normal lengths. Their angle bisectors therefore satisfy

$$2x - y = \pm(x - 2y),$$

which gives $x - y = 0$ and $x + y = 0$. Their combined equation is $x^2 - y^2 = 0$, so option **(A)** is correct.

Q4. (A)

Let the positive x- and y-intercepts be a and b. The line is $\frac{x}{a} + \frac{y}{b} = 1$, and passage through $(2, 3)$ gives $\frac{2}{a} + \frac{3}{b} = 1$. Put $u = 2/a$ and $v = 3/b$, so $u, v > 0$ and $u + v = 1$. The area of the triangle is $ab/2 = 3/(uv)$. Since $uv \le 1/4$, the area is minimised when $u = v = 1/2$, giving $a = 4$ and $b = 6$. Therefore the line is $\frac{x}{4} + \frac{y}{6} = 1$, or $3x + 2y = 12$. The answer is **(A)**.

Q5. (B, D)

Let m be the slope of the third side BC. Since the base angles at B and C are equal, the acute-angle tangents made by BC with the two given sides must have equal magnitudes:

$$\left|\frac{7 - m}{1 + 7m}\right| = \left|\frac{-1 - m}{1 - m}\right|.$$

Considering the two possible signs and simplifying gives either $m^2+1=0$, which has no real solution, or

$$3m^2+8m-3=0 \quad \Longrightarrow \quad (3m-1)(m+3)=0.$$

Thus $m=\frac{1}{3}$ or $m=-3$. The lines through $(1,-10)$ with these slopes are

$$y+10=\frac{1}{3}(x-1) \implies x-3y=31,$$

and

$$y+10=-3(x-1) \implies 3x+y+7=0.$$

Therefore options **(B)** and **(D)** are correct.

Q6. (A, C)

The distance between $2x+y=3$ and $2x+y=5$ is $\frac{2}{\sqrt{5}}$. If a transversal makes angle ϕ with the parallel lines, the intercepted segment has length $\frac{2/\sqrt{5}}{\sin\phi}$. Requiring the length to be 2 gives $\sin\phi = 1/\sqrt{5}$. Both the vertical line $x=2$ and the line $3x+4y-18=0$ satisfy this condition and pass through $P(2,3)$. The horizontal line has intercepted length 1, while $2x+y=7$ is parallel to the given lines. Hence **(A)** and **(C)** are correct.

Q7. (B, D)

Putting $y=mx$ in the homogeneous quadratic part gives

$$m^2-6m+a=0.$$

Its roots are m and m^2. Comparing their sum and product,

$$m+m^2=6 \quad \Longrightarrow \quad m^2+m-6=0,$$

so $m=2$ or $m=-3$. Hence $a=m\cdot m^2=m^3$, giving $a=8$ or $a=-27$. The correct options are **(B)** and **(D)**.

Q8. (A, C)

The two given lines are parallel, and their distance is

$$\frac{15}{\sqrt{4^2+(-3)^2}}=3.$$

Since $BC=2$, the area is $\frac{1}{2}\cdot 2\cdot 3=3$, so (A) is true. For the centroid $G=(X,Y)$, the linear expression $-4x+3y$ has value 15 at A and 0 at each of B, C. Therefore

$$-4X+3Y=\frac{15+0+0}{3}=5,$$

so $3Y=4X+5$ and (C) is true. The other statements do not hold for arbitrary placement of the segment BC.

Q9. (A, C, D)

Factor the equation: $6x^2 - 7xy + y^2 = (6x - y)(x - y) = 0$. The slopes are 6 and 1, so (A) is true and the lines are not perpendicular, so (B) is false. Their angle satisfies

$$\tan\theta = \left|\frac{6-1}{1+6}\right| = \frac{5}{7},$$

so (C) is true. The factorisation proves (D) as well.

Q10. 7

Let $Q = (x, y)$ be either endpoint of the chord. Using $x^2 + y^2 = 100$ and $PQ = 4$ with $P = (8, 6)$,

$$(x-8)^2 + (y-6)^2 = 16 \quad \Longrightarrow \quad 8x + 6y = 92.$$

Therefore the chord is $4x + 3y = 46$, so $a = 4$, $b = 3$ and $a + b = \boxed{7}$.

Q11. 5

Write $A = (13\cos\theta, 0)$ and $B = (0, 13\sin\theta)$, since $AB = 13$. The unit vector from A to B is $(-\cos\theta, \sin\theta)$. In the 5–12–13 triangle, $\cos A = 12/13$ and $\sin A = 5/13$. Taking the configuration with C inside the triangle,

$$\overrightarrow{AC} = 12\left(-\frac{12}{13}\cos\theta + \frac{5}{13}\sin\theta,\ \frac{12}{13}\sin\theta + \frac{5}{13}\cos\theta\right).$$

Adding this vector to A gives

$$x_C = \frac{25\cos\theta + 60\sin\theta}{13}, \qquad y_C = \frac{60\cos\theta + 144\sin\theta}{13} = \frac{12}{5}x_C.$$

Hence the locus is $12x - 5y = 0$, so $k = \boxed{5}$.

Q12. 3.16

For the stated choice of D, the configuration yields the line $AD : 3x + y - 10 = 0$. The distance from the origin to this line is

$$\frac{|-10|}{\sqrt{3^2 + 1^2}} = \frac{10}{\sqrt{10}} = \sqrt{10} \approx \boxed{3.16}.$$

Q13. 2

The quadratic part factors as

$$3x^2 - 5xy - 2y^2 = (3x + y)(x - 2y).$$

The candidate factor through the origin is either $3x + y = 0$ or $x - 2y = 0$. For $x - 2y = 0$, the second intersection with the circle is $R = (28/5, 14/5)$, and at $P = (0, 6)$ the angle satisfies $\tan\angle OPR = (28/5)/(16/5) = 7/4$, as required; the other line does not satisfy the angle condition. Hence the required factor is $x - 2y$. For the full expression to split into a pair of lines with this factor, write

$$(3x + y + p)(x - 2y) = 3x^2 - 5xy - 2y^2 + px - 2py.$$

Comparing the coefficient of x with the given expression gives $p = -1$, and comparison of the coefficient of y gives $k = -2p = 2$. Therefore $k = \boxed{2}$.

Q14. 1

Factor the pair:

$$x^2 - 5xy + 6y^2 = (x - 2y)(x - 3y) = 0.$$

The slopes are $m_1 = \frac{1}{2}$ and $m_2 = \frac{1}{3}$. Thus

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1/6}{7/6} = \frac{1}{7}.$$

Hence $7\tan\theta = \boxed{1}$.

Q15. 2

For L_t to be parallel to $3x + 4y - 7 = 0$, the ratios of the coefficients of x and y must agree with $3 : 4$. Thus

$$4(1 + 2t) = 3(2 + t) \quad \Longrightarrow \quad 4 + 8t = 6 + 3t \quad \Longrightarrow \quad 5t = 2.$$

The required answer is $\boxed{2}$.

Passage 16

16.1 (A): Solving the two generating lines $2x + y + 4 = 0$ and $x - 2y - 3 = 0$ gives $I = (-1, -2)$.

16.2 (B): The slope of IM is $\frac{-3-(-2)}{2-(-1)} = -\frac{1}{3}$, so the perpendicular member has slope 3. Its equation through $I = (-1, -2)$ is $y + 2 = 3(x + 1)$, or $3x - y + 1 = 0$. This is the member obtained when $a = b$, so the ratio is $1 : 1$ (option B).

Passage 17

17.1 (B): The intercepts of L_t are

$$a_t = \frac{6}{t + 1}, \qquad b_t = \frac{6}{2 - t}.$$

For equal intercepts, $a_t = b_t$, so $t + 1 = 2 - t$ and $t = \frac{1}{2}$. Hence option **(B)** is correct.

17.2 (A): The slope of L_t is $-\frac{t+1}{2-t}$. The line $2x - y + 1 = 0$ has slope 2, so a perpendicular line must have slope $-\frac{1}{2}$. Thus

$$-\frac{t + 1}{2 - t} = -\frac{1}{2} \quad \Longrightarrow \quad 2(t + 1) = 2 - t \quad \Longrightarrow \quad t = 0.$$

Therefore option **(A)** is correct.

Q18.

(A) $x^2 - 5xy + 6y^2 = (x - 2y)(x - 3y)$ has slopes $1/2$ and $1/3$, so $\tan\theta = 1/7$ and (A)$\rightarrow$(r).

(B) $3x^2 - 10xy + 3y^2 = (3x - y)(x - 3y)$ has slopes 3 and $1/3$, so $\tan\theta = 4/3$ and (B)$\rightarrow$(s).

(C) $x^2 - y^2 = (x - y)(x + y)$ represents perpendicular lines, so (C)$\rightarrow$(q).

(D) $(x + y)^2 = 0$ is a repeated line, so (D)$\rightarrow$(p).

Q19.

(A) The generating lines $x + y - 2 = 0$ and $x - 2y + 1 = 0$ meet at $(1, 1)$, so (A)$\rightarrow$(p).

(B) For $y = 2mx - m^2$, the envelope condition $0 = \partial y/\partial m = 2x - 2m$ gives $m = x$; substituting yields $y = x^2$, so (B)$\rightarrow$(q).

(C) The family $x\cos\theta + y\sin\theta = 2$ consists of tangent lines to the circle of radius 2 centred at the origin; its envelope is $x^2 + y^2 = 4$, so (C)$\rightarrow$(r).

(D) Every member of $a(x-1)+b(y+2) = 0$ passes through $(1, -2)$, so (D)$\rightarrow$(s).